Abstract—A basic tenet of special relativity is the concept of length contraction seen by an observer in motion. Lorentz contraction, which changes the apparent location of a light source, combines with aberration, which changes the apparent direction to the source, producing a variety of effects. While aberration has been confirmed, Lorentz contraction has never been tested directly, due to the generally negligible size of the effect. As the earth orbits the sun, Lorentz contraction offsets the apparent position of a distant source by as much as 18 micro-arcseconds (µas) per degree of separation. This offset is in addition to that caused by aberration. The Space Interferometry Mission, due for launch in 2005, promises a resolution of +/- 1 µas in a field of view of one degree, allowing for the first time the direct confirmation of Lorentz length contraction, one-hundred years after the introduction of Einstein’s special theory of relativity in 1905.

1. INTRODUCTION

According to the Lorentz transformations as applied to special relativity, there is time dilation and length contraction in any system moving with respect to the observer. This transformation represents an actual change in the dimensions of length and of time, not just a shortening of rulers or a slowing of clocks, and not just a visual effect. The basis for these transformations is Einstein’s second postulate, the assumed constant velocity of light from any given source as measured from all inertial frames of reference. In order to support this hypothesis, and maintain a mathematically consistent world-view, the Lorentz transformations must be invoked. It is impossible to test the invariance of the velocity of light directly, for the simple fact that no two observers can absorb the same photon. Though we know the velocity as measured with respect to an observer that detects a particular photon, we can never know the velocity of that particular photon with respect to any other observer. The same is true of any photon detected by any observer in any reference frame—there is a unique detection by one and only one observer of any given photon. As a result, we are left with testing for the implications of the second postulate. The most basic secondary effects to test for are length contraction and time dilation.

As simple as this seems, there has never been any direct test of either effect. First consider the case of time dilation. Special relativity predicts that time runs slowly in any inertial frame moving with respect to the frame of the observer. Testing this effect requires two identically constructed and calibrated clocks. Each clock must be constructed and calibrated in the appropriate reference frame for the test. Thus each clock’s reference frame of rest must already be in motion with respect to the other clock’s reference frame during the construction process. It has been demonstrated in another paper that any two such clocks, constructed and calibrated by the same methodology in relatively moving inertial frames will maintain synchronous time [1]. Such a test, including the problem of finding two inertial frames, is extremely difficult and has never been performed.

Instead, we have relied on constructing identical synchronous clocks in the same reference frame. One of these clocks is then moved into another reference frame, and the rates of the clocks are compared, either directly or by comparing the accumulated time on each. The process of moving the clock out of one reference frame and into another of necessity involves the application of acceleration, and thus energy, to the clock system, whether it be a muon or an atomic clock. These clocks do indeed slow down, but it is impossible to determine between any of several possible reasons for this effect. The result may be due to the application of energy to the system (as this author has demonstrated is probable), it may be due to an empirical change in time recording mechanisms between reference frames, or perhaps the dimension of time itself is actually skewed in a manner predicted by the Lorentz transform.

Next we look at length contraction. While the tests of time dilation are weak and inconclusive, there has never been
any direct test of the Lorentz length contraction. The reason for this is quite simple. In order to test directly for length contraction, we need a combination of very high speeds and very precise measurements. Additionally, any occurrence of length contraction invariably includes aberration, a much larger effect. The aberration effects must be separately accounted for in order to isolate the length contraction effects. Even as the twentieth century draws to a close, there is no way to test directly for the Lorentz length contraction. But such a test will soon be possible.

In 2005, the National Aeronautics and Space Administration (NASA) will launch the Space Interferometry Mission (SIM). This satellite uses a highly stable platform combined with a very precise interferometer to measure the angular separation of stars and galaxies. This project promises unprecedented accuracy in terms of astrometric grid calculations and star mapping. The sensitivity of the mission is +/- 1 microarcsecond (mas) in a field of view of one degree, and +/- 4 mas in a field of view of fifteen degrees. This resolution is less than the width of your finger as seen from the moon. The next best star mapping ever performed has been by the HIPPARCOS satellite, at a precision of only milliarcseconds (mas). To understand the importance of this increased sensitivity, we must first look at an effective approach to measuring length contraction.

2. LENGTH AND SUBTENDED ANGLES

In the spirit of Einstein, in Figure 1, we see a high-speed train and another observer stationary on the embankment. In the extreme distance is a uniformly spaced set of equal height telephone poles. The distance from the tracks to the poles is very large, and measured in advance to be a certain value \( r \). Under special relativity, this distance will not change for the train observer, as it is always normal to the direction of the train’s motion, at least for poles tightly clustered around the \( y \) axis. The Lorentz transformations are such that the effect on length occurs only in the direction of motion. Thus, if we assume the train is moving along the \( x \) axis, then the distance to the poles lies along the \( y \) axis, and the height of the poles lies along the \( z \) axis. Neither of these dimensions will change. However, the distance between each set of poles is measured along the \( x \) axis and will change according to the Lorentz length contraction formula.

If the poles are far enough in the distance, they will not appear to move in the field of view of the train rider. This observer can make very precise measurements of the angle of separation between any two poles. Since the value of \( r \) was determined in advance, the observer can calculate a value for the distance between the poles as follows:

\[
\sin \theta_x' = \frac{d}{r} \quad (1)
\]

\[
d = r \sin \theta_x' \quad (2)
\]

The train rider can also determine the height of the poles by a similar method, such that:

\[
h = r \sin \theta_y' \quad (3)
\]

The observer on the embankment can make similar angle measurements and thus determine the separation distance and height as seen from the embankment frame of reference. Clearly, any change in \( d \) produces a proportional change in \( \theta_x' \). Now we can introduce the effects predicted by special relativity.

![Figure 1 Length and Subtended Angles](image)
3. ABERRATION

The first effect to consider is aberration. Aberration, as a local effect applicable to the observer, is a well documented phenomenon, and is not unique to the theory of special relativity. However, the means by which aberration arises in special relativity is important, and will be addressed in section 5. For the moving train rider, all angles measured along the $x$ axis will be visually precessed by the aberration factor. The aberrated position of a star on the $y$ axis will be moved forward through an angle such that:

$$\sin \theta_a = \frac{v}{c}$$ (4)

This skewing of angles due to aberration results in a change in the apparent angular separation between any two distant objects lying apparently parallel to the $x$ axis. As can be seen from Figure 2, the apparent distance, $d'$, between any two objects is reduced as below:

$$d' = d (1 - \frac{v^2}{c^2})^{1/2} = d \gamma^{-1},$$ (5)

where we define $\gamma = (1 - v^2 / c^2)^{1/2}$.

Substituting (5) into equation (1) we get:

$$\sin \theta'_a = \frac{d'}{r} = \frac{d}{r} \gamma^{-1} = (\sin \theta_a) \gamma^{-1}$$ (6)

For small angles, the $\sin$ of the angle changes in direct proportion to the angle itself, and we can approximate:

$$\theta'_a \approx \theta_a \gamma^{-1}$$ (7)

While the above approximation is useful for conceptual studies, in any real observations the actual angles would need to be computed directly from the $\sin$ values, or vice versa to determine the true effects.

In the above we have seen that the aberration effect produces an apparent length contraction in the direction of motion of the moving observer. For the specific case considered, that of an object lying physically on the $y$ axis, the value of this apparent contraction is even the same as that predicted by Lorentz contraction, namely $\gamma^{-1}$. However, this effect is not Lorentz length contraction.

Imagine standing perfectly still and straight, so that we can define the line joining your shoulders as the $x$-axis. If you hold a ruler directly in front of you at arm’s length, oriented parallel to the $x$-axis, it will appear to have a certain length, or subtended angle, in your field of view. If you now rotate your arm to the left or right, being careful to keep the ruler aligned along the $x$-axis, the apparent length, or subtended angle, will become smaller. This foreshortening of objects rotated through an angle is the effect we see in aberration. In this example, its value is coincidentally the same as that predicted by Lorentz contraction, and in fact, it combines with Lorentz contraction to produce an even greater overall observed effect. As with the case of length contraction, referring again to Figure 1, we see that aberration will affect the apparent distance between the poles, but will have no effect on the apparent height of the poles.

![Figure 2 Aberration Contraction](image)

4. LORENTZ LENGTH CONTRACTION

According to Einstein, we can measure the length of a train from an embankment by the following procedure. We use several observers stationed along the embankment to mark the location adjacent to the front of the train and the end of the train at some particular time measured in the reference frame of the embankment observers. After the train has moved on, we measure the distance between these two marks with a meter stick. According to special relativity, this measured distance will be less than the length of the train as measured by a rider on the train with an equivalent meter stick. The train’s length measured by the
embankment observer will be shortened by the factor $\gamma'$ as shown:

$$l' = l(1 - \frac{v^2}{c^2})^{1/2} = l\gamma^{-1}$$

(8)

Note that in the above, the local visual effect of aberration plays no role. The embankment observers can be placed arbitrarily close to the train, even in contact with the train via mechanical/electrical contacts, and the length of the train can in theory be made arbitrarily large to offset any visual effects. An embankment observer located some distance away from the train would see the contacts marking the end points of the moving train at the smaller angle subtended by the length contracted train, as in figure 3. In the figure, the same train, stationary on the track, subtends a larger angle in the distant observer’s field of view. The moving length-contracted train fires a different set of contacts, and the angle subtended between these contacts from the observer’s position is smaller. Since the observer and the contacts are in the same reference frame, aberration plays no role in creating the contracted angle. Any visual length contraction effects due to aberration would thus apply on top of the actual length contraction of the train.

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Using Einstein’s example of mechanical/electrical contacts located along the embankment, the embankment observers would measure a length contracted train along the x axis, but the height of the train measured along the z axis would remain unchanged. Similarly, according to the Lorentz transformations, both observers in Figure 1 will determine the same value for the height, $h$, of the poles, independent of the effects of Lorentz contraction or aberration. However, ignoring aberration for the moment, the train rider should measure the distance between the poles as shortened by the Lorentz contraction. To do this, the train rider must first assess and account for the local effects of aberration. Any residual lessening of $\theta$ and thus $d$ is attributable to Lorentz contraction. After backing out the aberration effects, this observer’s value for the Lorentz contracted distance between the poles compared with the embankment observer is exactly that expressed in (8).

Using the same reasoning as we did for aberration, we see that the modified subtended angle due to length contraction alone is reduced proportionally to the amount of length contraction. The effective physical angular separation of the poles after length contraction is:

$$\sin \theta'_x = \frac{d'}{r} = \frac{d}{r\gamma^{-1}} = (\sin \theta_x)\gamma^{-1}$$

or $\theta'_x = \theta_x\gamma^{-1}$

(9)

The local visual effect of aberration will be applied with respect to light arriving from the new positions of the poles obtained due to the real Lorentz length contraction.

5. ABERRATION VS. LORENTZ CONTRACTION

We must look at the cause of aberration in special relativity closely to see that this observed effect is independent of, and in addition to, the Lorentz length contraction. Aberration occurs independently of the motion of the source, and is an effect established locally by the velocity addition formula as applied to the motion of light from the source with respect to the motion of the observer.

For simplicity, assume that light is coming in at some angle $\alpha$ measured with respect to the x-axis in the x-y plane for an observer stationary in Figure 4. An observer moving at a constant velocity along the x-axis will observe the light arriving at the aberrated angle $\alpha'$. In the stationary observer’s frame, light has the following velocity components:

$$u_x = -c \cos \alpha \quad u_y = -c \sin \alpha$$

(8)

For the moving observer, the velocity components of the oncoming light are:

$$u'_x = -c \cos \alpha' \quad u'_y = -c \sin \alpha'$$

(9)
The relativistic velocity addition formula gives the relation between the primed and unprimed components:

\[ u'_{x} = \frac{u_{x} - v}{1 - u_{x}v/c} \]

\[ u'_{y} = \frac{u_{y}}{\gamma(1 - u_{y}v/c^{2})} \]  

(10)

Substituting (10) into (8) gives the usual aberration formula:

\[ \cos\alpha' = \frac{\cos\alpha + v/c}{1 + (v/c)\cos\alpha} \]  

(11)

This important formula is derived without considering the position of the source. The velocity addition formula is developed straightforwardly from applying length contraction and time dilation between frames. The angle of aberration of incoming light is then determined from a simple application of the velocity addition formula [5].

For light from a given arbitrary source, we can easily determine the relation between angles of incidence among several observers in relative motion. That position information about the source of light is missing is significant to the problem at hand. Aberration affects the observed direction of an incoming photon from a particular source. Lorentz contraction alters the actual, physical location of the source, prior to the photon’s leaving it.

Aberration is therefore clearly a local phenomenon. Light from a distant star could be heading toward Earth for a billion years, say from due north. As it is viewed on Earth, it will be aberrated one way or another, depending on the particular season it reaches us. Due to the extreme distance of the star, aberration will cause the apparent line-of-sight to betray the star’s “true” position by hundreds to thousands of light years. The source is certainly not jumping around like this, and the light must be coming directly from the actual source location to the Earth along the line joining the location of star and Earth, not along the aberrated line of sight. If light left the source at the aberrated angle from which it appears to be reaching us, it would miss the Earth by hundreds to thousands of light years. The dished line of Figure 4 shows the effect of light leaving the source at the aberrated angle—such light never reaches the observer.

It could be argued that the length contraction and time dilation of special relativity are purely observational effects. One could say that the only means by which these two effects manifest themselves is by visually observing objects as they move away from us or towards us. Since these objects are always on the move, one cannot say whether the effect is real, or purely observational. But this line of reasoning quickly fails, and has been demonstrated to be false in a variety of settings, as we see below.

When a clock is placed in motion with respect to its original rest frame, the clock actually slows down. We know the clock slows down because it accumulates less time while in motion. If the moving clock is ultimately returned to its point of origin, the elapsed time physically displayed on the clock that was moving will be less than the elapsed time on the laboratory clock, even though they are now side by side in the same reference frame. This was demonstrated inconclusively by Hafele and Keating [6], but has been demonstrated to an unprecedented level of accuracy in the Global Positioning Satellite system. Each of the satellite clocks is pre-corrected for the effects of its orbital velocity prior to launch.

If the slowing of these clocks due to motion is the result of relativistic time-dilation, the effect is clearly real, as anticipated by Einstein, and is not simply an observational effect. If relativistic time-dilation is a physically detectable event, then relativistic length contraction must be physical as well, as anticipated by Einstein’s train-length measurement proposal. One of the difficulties in Lorentz’s original contraction theory was that he considered the length contraction to be real, while the time dilation was a mathematical artifact of no real significance. It would present an identical problem for special relativity to claim that time-dilation is real but that length contraction is simply a visual effect with no physical basis.

We can also consider the case of muons entering the Earth’s atmosphere. These particles travel the distance from the upper atmosphere to sea level in the course of an average muon lifetime. Even at velocities approaching c, the distance traveled by these particles would require the average life of a muon to be several times its rest value. As we stand on Earth, we can explain the muons’ ability to reach sea level as being due to time dilation. Since the particles are moving very fast, their internal clocks have slowed, causing their average life span to increase several fold. With a longer life, it is easy for them to finish the journey before they decay.

In the muons’ frame of reference, the situation is quite different. The only way this can happen in the muons’ reference frame is if the actual physical distance that must be traveled by them is shortened as in (8). This is not a visual effect for the muon. If the distance traveled by the muon is not physically shorter, the muon simply does not remain in existence long enough to make the trip, even at speeds greater than .9c. To the muon, length contraction is clearly not merely a visual effect, as the muon is not “seeing” anything. The distance to be traveled by the muon from the upper atmosphere to sea level is physically shorter than the same distance measured by a slower moving particle. The high speed muon performs Einstein’s train embankment experiment first hand.

As the SIM spacecraft follows the Earth in its orbit about the sun, its situation is indistinguishable from that of the high-speed muon. In the reference frame of the SIM, lengths must be physically contracted in the direction of motion, whether or not the SIM is “seeing” anything. To an observer at solar barycenter, no length contraction would
occur, but the SIM clocks would be running slowly instead. However, we are interested only in what happens in the reference frame of SIM, and in that reference frame, as with the muon approaching sea level, all lengths are contracted in the direction of motion as compared to the solar barycenter frame, which we will use as our “stationary” reference.

6. ABERRATION AND LORENTZ CONTRACTION IN ASTROMETRICAL OBSERVATIONS

We can now determine the entire effect, Lorentz contraction combined with aberration, by consulting Figure 5. In Figure 5, with the earth moving in the same direction as the apparent line joining the two objects, 0 represents the change in subtended angle of a line drawn to the source due to length contraction, while 0' represents the combined line-of-site visual effect of aberration with Lorentz contraction.

Combining equations (9) and (7), we have:

\[
\sin \theta'' = (\sin \theta') \gamma^{-1} = (\sin \theta) \gamma^{-2}
\]

or, for conceptual approximation purposes for small angles:

\[
\theta'' \approx \theta \gamma^{-2}
\]

Consider a set of stars or galaxies appearing almost overhead from the plane of the earth’s orbit about the sun. As the earth orbits the sun, its velocity along a line joining any two such objects will change from a minimum of 0 km/sec to a maximum of about 30 km/sec every three months. The effective aberration angle along this line will also change due to the combined effects of aberration and Lorentz contraction as in (15). The change in subtended angle will vary from a maximum to a minimum every three months as well. We can calculate the magnitude of this effect for the velocity of the earth, where \(\Delta \theta\) represents the change in subtended angle due to length contraction, and \(\Delta \theta_a\) represents the change in subtended angle due to aberration:

\[
\frac{\Delta \theta}{\theta} \approx \frac{\theta - \theta^{-1}}{\theta} = 1 - \gamma^{-1}
\]

\[
1 - \gamma^{-1} \approx 5 \times 10^{-9}
\]

Lorentz Contraction: \(\frac{\Delta \theta}{\theta} \approx 5 \times 10^{-9} \text{deg/deg} \approx 18 \mu\text{as/deg}
\]

Aberration: \(\frac{\Delta \theta_a}{\theta} \approx 18 \mu\text{as/deg}
\]

Combined Effect: \(\frac{\Delta \theta}{\theta} \approx 36 \mu\text{as/deg}
\]

As the earth orbits the sun, the angular separation of stars and galaxies should vary by as much as +/- 36 \(\mu\)as per degree of field of view. This variation, equal to only 1 part in \(10^8\) is very small, well below conventional means for detection. But the means will soon be available.

7. NASA’S SPACE INTERFEROMETRY MISSION

In 1886, Michelson and Morley performed a test for the earth’s motion through the presumed aether using two orthogonal beams of light and the interference pattern obtained. This device, referred to as a Michelson interferometer, almost single handedly eliminated the concept of an all pervading aether from the minds of most physicists of the day, and led the way for the acceptance of Einstein’s special theory of relativity nine years later.

The same concept, observing the shift in interference patterns produced by divergent and recombined light paths from the same source, has been gaining popularity in the world of astronomy. From the Very Large Baseline Interferometer (VLBI) array to the recent space-based HIPPARCOS mission, unprecedented resolution in visibility, position and proper motion measurements have been made. The final results of the HIPPARCOS mission provided star tables with a median standard error in position and proper motion measurements on the order of 0.8 mas from a defined grid. The effects predicted in this paper are much smaller than the resolution of HIPPARCOS and would not appear in the overall results from that program, the best to date. When fully operational, the U.S. Navy’s ground-based Prototype Optical Interferometer (NPOI) promises a resolution of 200 \(\mu\)as. This array of six telescopes is incredibly useful for observations of orbits of double stars and for planet hunting, but still does not possess the resolution required for a direct test of Lorentz contraction. As addressed in the Introduction, the length
contraction portion of the overall aberration effect has to date never been an issue due to the negligible size of the effect on overall observations.

In 2005, NASA plans to launch the Space Interferometry Mission (SIM) satellite. SIM operates by comparing the change in path length along two arms of an interferometer of light from a test star compared to a baseline or grid star. By measuring this change in path length to an accuracy of 1 nanometer the position of the test star can be obtained within about 4 μas of the grid star. This level of accuracy can be obtained over the entire field of view of the instrument, which is about 15 degrees. In a field of view of 1 degree or less, the predicted accuracy is on the order of 1 μas. The changes predicted by relativistic aberration and length contraction, illustrated in an exaggerated manner in Figure 6, amount to almost 540 μas in a field of view of 15 degrees every three months, more than 100 times the proposed resolution capability of the instrument.

![Figure 6 Observed Effects of Length Contraction](image)

Among the requirements currently placed on SIM is the ability to point everywhere on the celestial sphere outside of a 50 degree sun exclusion area during a thirty day period. If an observation were to be made of a particular pair of objects at approximate zenith at one point in time, then that same pair of objects should be available for viewing again after the desired 90 day interval, +/- 15 days. Thus the capability, opportunity and required precision exist in SIM to perform the direct test of Lorentz length contraction proposed in this article. The next section provides a more detailed discussion of the topics presented herein.

8. DISCUSSION

The literature has addressed several aspects of the search for a measurable Lorentz contraction, [2,3,4] for example. There are many different aspects to this problem, each of which must be considered for the specific case under consideration. These issues include:

- Are we viewing a three dimensional object, or simply measuring the apparent angle between two points
- Are the objects or points close together, or do they subtend a large angle
- Are we close to the objects or extremely far away
- Does aberration nullify or combine with length contraction for the particular angles involved
- Do the points under consideration move in our field of view due to our motion during the instant of observation (other than aberration effects)
- Does the time of emission play a role (e.g. millisecond pulsars), or are we interested only in the position of a continuous source
- Is parallax shift adequately addressed and modeled

For example, if we are viewing a small cube or a solid, three dimensional bar at a large distance, the cube will appear to be rotated, even though it is not. If we measure the length of the sides by the angle subtended, they will be shortened, but since the cube appears rotated, the amount of Lorentz shortening will exactly match the amount of expected foreshortening due to the apparent rotation, and the impression on our observations will be that the Lorentz contraction has not occurred. Similarly, a spherical object will still appear spherical after applying aberration and length contraction, but there will be a change in scale due to Lorentz contraction.

In the following set of figures, we will trace the apparent and actual positions of several stars in space and on the observer’s visual sphere as they are affected by Lorentz contraction and aberration. In Figure 6 we see the stars as they appear in some baseline reference frame, say the solar barycenter frame, which for convenience we will call the stationary reference frame. The size of the visual sphere will be seen to be immaterial except as a convenient tool for displaying the degree of aberration.

Independently of the actual location of the stars, the light that ultimately reaches an observer comes in a finite amount of time from a sphere whose radius is c times that finite amount of time. If we assume a time of one second, then the radius of the visual sphere is one light second.

In figure 7 we have labeled two stars A and B, and have measured their angular separation visually to be 45 degrees in the solar barycenter frame. The stellar objects we are viewing are all sufficiently far enough away that they will not appear to move from their position in the sky over any arbitrarily small incremental viewing time span.

When we drive in a car and look out the side window, trees and light poles move in our field of view, but the sun and moon appear fixed for long periods of time due to their extreme distance. Only if we travel a significant distance will these items move appreciably in our field of view.
Thus, as the Earth orbits the Sun, and changes its velocity with respect to the far field of stars, the only effects we must consider at each incremental viewing are aberration and length contraction. There will be a parallax shift due to the overall change in the Earth’s position in its orbit with time, but this large scale effect is adequately covered in the SIM algorithms, and will not be discussed here.

In Figure 8 we see the application of length contraction on the stars position in space due to a velocity of our reference frame of \(0.5c\) in the x-direction with respect to the prior assumed stationary frame. This is the result of the physical length contraction discussed by Einstein in his railroad embankment example. It exists independently of the aberration effects. The positions of the stars after length contraction are projected on a visual sphere centered instantaneously on the moving observer. Clearly, the visual
sphere must be a uniform sphere in the observer’s reference frame by definition.

The points at which the projections hit the visual sphere represent a virtual source. It is from this virtual source that we must apply the aberration calculations, as we will see shortly. The length contractions illustrated in figure 8 take into account only the motion of our reference frame to that of the presumed stationary frame. It does not address the proper motions of the stars or galaxies being viewed, as these have no discernable effect.

If a galaxy has its own large proper motion with respect to our presumed rest frame, that motion will not change due to the periodic motion of the Earth. The combined length contraction effect of the galaxies proper motion plus that of the Earth is, to first order in \( v^2/c^2 \) simply the sum of the two values, with the component due to proper motion representing a constant offset for all Earth velocities. The secondary effects are on the order of \( v^4/c^4 \), and are two small to be considered, even in SIM. The galaxies proper motion also plays no role in aberration, since aberration is a local visual effect based on relative change in the observer’s motion only. The only effect the galaxies own proper motion has with respect to length contraction is that the angle subtended from one end of the galaxy to the other in the direction of motion is foreshortened. However, since the proper velocity of the galaxy remains basically constant with time, the apparent visual size of the galaxy due to its own motion also remains constant, and thus plays no role in our observations.

In Figure 8, we see that the angular separation between stars A and B has been reduced due to length contraction. These virtual sources are labeled A’ and B’ in the figure. Their separation is now only about 41 degrees. However, one cannot physically observe the stars in this position, as they will be aberrated along the visual sphere, due to the same motion of the observer’s reference frame that produced the length contraction.

In order to apply the aberration calculation, we must begin with the angular position of each virtual source created by the length contraction in Figure 8, not the angle of the uncontracted virtual source of Figure 7. The aberration of the contracted position virtual source occurs due to the velocity addition formula applied to the motion of light from the virtual source reaching the moving observer, as we saw in section 5.

Figure 9 applies the aberration formula of equation (11) to each virtual source in the figure. With the exception of the two stars on the x-axis, each virtual source is aberrated toward the direction of motion of the observer. The greatest angle of aberration occurs for the star located directly on the z- (or y-) axis.

The aberrated, contracted position of the two stars A and B actually viewed by the observer are labeled A” and B”. The angular separation ultimately measured by the moving observer is only about 30 degrees. If we consider aberration alone without including length contraction, this moving observer would see an angular separation of about 33 degrees. Failing to include Lorentz contraction in this example reduces the sensitivity of our observations by 3 degrees per 45 degrees of field of view at a velocity of 0.5c.

The examples presented in the figures are for an observer velocity of .5c, much greater than the velocity of the Earth, which changes by 30 km/sec in value and direction every three months. A change in velocity of this magnitude would
not even begin to manifest itself in the figures presented herein. However, with a resolution of 1 \(\mu\)as/degree, SIM will be able to see the effects even for such a relatively small change in velocity. Failure to include Lorentz contraction in the SIM data reduction algorithms will reduce the sensitivity of observations by as much as 18 \(\mu\)as per degree of field of view. This error will render the SIM goal of 1 \(\mu\)as per degree unattainable.

9. CONCLUSIONS

In 1919, Sir Arthur Eddington made observations of the angular displacement of stars during a solar eclipse. Newtonian theory predicted one value, while general relativity predicted exactly twice that value. As is the case today, the effects to be observed were at the extreme limits of available technology. When Eddington’s observations appeared to come down on the side of general relativity, a New York Times headline proclaimed: “New Theory of the Universe. Newtonian Ideas Overthrown.”

Special relativity predicts that if we look at two stars with an angular separation of about 1 degree at time intervals of ninety days, we will see a variation in the observed angle of separation. In the case of stars directly overhead (and several other key angles), this variance is made up of two equal components. The first component is a variation of 18 \(\mu\)as due to Lorentz length contraction. The second component is a variation of 18 \(\mu\)as due to aberration. The SIM promises an angular resolution of 1 \(\mu\)as in a field of view of 1 degree, more than thirty times better than the effects predicted. If the combined effect of relativistic length contraction with aberration is not included in the data reduction algorithms for SIM, then it will not be possible to adequately obtain the sought for precision. Over the course of three months, the error in location of even the grid stars against which all measurements are ultimately compared will vary by as much as 18 \(\mu\)as, making the overall resolution of the grid less than the desired resolution of measurement.

This paper has discussed generally the nature of the relation between the Lorentz contraction of the dimension of length in the direction of motion and the local aberration of light from the position- contracted source, or virtual source. The discussion has considered the case of a single observer located at a point. SIM is a stereoscopic instrument, with a baseline of roughly 10 meters. The entire analysis presented herein must be carried out for both ends of the interferometer to determine what additional impacts, if any, this stereoscopic view will have on the received phase of light at each window during seasonal changes in spacecraft velocity. It is unlikely that including length contraction should prove any more troublesome than the solution for aberration, and simply requires a reassessment of those algorithms.

An added benefit of the level of precision inherent in SIM is that SIM can determine the angle of aberration itself to an unprecedented degree of accuracy. Using this measured angle of aberration, the angular displacement effects of that aberration may be calculated directly. Any residual effect is then due to Lorentz contraction. As in 1919, relativity predicts a value twice that to be otherwise expected. If the residual is found, then special relativity will, for the first time, have passed a direct test of one of its most fundamental predictions. If the residual is not found, then special relativity may have to be abandoned and the New York Times may wish to consider a retraction.

REFERENCES


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