ABSTRACT

Special relativity (SRT) was born on the basis of a gedanken experiment involving the relative simultaneity of distant events as perceived by observers with different inertial velocities. It is this assumed aspect of special relativity that is most troubling to one’s intuition, accustomed as we are to living in a world of absolute, not relative, simultaneity. Regardless of the adequacy of special relativity to present a true model of the nature of space and time, the theory at least presents a mathematical equivalence to most problems to which it is applied. Such tests include Doppler effects, clock retardation and apparent mass increase with velocity. As such, further tests of these effects to even greater precision are not likely to produce any new insights into the validity of special relativity. Surprisingly, however, an actual test of the most troubling aspect of SRT—relative simultaneity—has already been performed, and demonstrates that relativistic simultaneity, in the form of the relativistic velocity addition formula, is incorrect.

FRESNEL’S COEFFICIENT OF AETHER DRAG

Fresnel was a firm believer in the concept of an aether, as it was “establishment” physics of the day. In 1818 he proposed that the density of aether in a material body was different than that in the free aether. From an application of elastic waves to optics, Cauchy demonstrated that the ratio of the elastic constant, \( \rho \), of a substance to \( \rho_1 \), the measure of its density, or mass per unit volume, is equal to \( c_1^2 \), or:

\[
\frac{\rho}{\rho_1} = c_1^2 \quad (1)
\]

Building on this, Fresnel imagined a bar of cross sectional area \( F \) moving through the aether with velocity \( v \), as in Figure 1. This bar sweeps up aether, where it acquires a new density, \( \rho_1 \), yet is only partially carried along by the bar. Thus the velocity of the bar with respect to the constrained aether, \( v_1 \), is different than the velocity of the bar with respect to the outside aether. While the velocity and density of the constrained aether changes, its mass must remain constant. Thus, if the bar moves for a time, \( \tau \), we can state:

\[
\rho_1 F v_1 \tau = \rho F v \tau \quad \text{or} \quad v_1 = \frac{\rho}{\rho_1} v \quad (2)
\]

Comparing (2) and (1), we see that the velocity of the constrained aether with respect to the bar is simply:

\[
v_1 = \frac{\rho}{\rho_1} v = \frac{c_1^2}{c_2^2} v = \frac{c_1^2}{c_2^2} \frac{1}{\eta^2} v \quad (3)
\]

where the last term holds since the refractive index, \( \eta \), is defined as \( c/c_1 \), with \( c_1 \) representing the velocity of light through the bar at rest.

Figure 1. Fresnel’s Aether Hypothesis

Now, the velocity of light in the moving bar is simply the velocity of light through the aether in the bar minus the velocity of the constrained aether thought the bar, or:

\[
c_1' = c_1 - v_1 = c_1 - \frac{1}{\eta} v \quad (4)
\]

Finally, the velocity of light as measured by an observer stationary in the lab frame of the moving bar is given by \( c_1' \) plus the velocity of the bar through the lab frame, or:

\[
c_1'' = c_1' + v = c_1 + v(1 - \frac{1}{\eta^2}) \quad (5)
\]

This is the well-known convection coefficient, or “aether drag” coefficient as derived by Fresnel. Most books on relativity praise Fresnel for his insight and ability in obtaining this important formula, even though they contend he got there by using bad assumptions. His theory is considered to be what Petr Beckmann refers to as a mere “equivalence,” a theory that produces the correct answer mathematically but is based on the wrong underlying assumptions. [4]

Special relativity derives this same result, based on the relativistic addition of velocities as obtained by invoking length contraction and time dilation. This equation has been “verified” experimentally, though, in all cases, the desired result was known in advance. Later in this paper we will see how the proper application of the relativistic velocity addition theorem actually produces results inconsistent with experiment. But first we will show that these “verifications”
also support the Galilean view, well within the limits of experimental error, leading to the firm possibility that both Fresnel’s theory and Einstein’s theory are mere equivalences.

THE EXPERIMENT OF FIZEAU

In 1851, Fizeau carried out an experiment that tested for the aether convection coefficient. This was the first such test of Fresnel’s formula, derived without experimental evidence, over twenty years earlier. Fresnel, in fact, had died more than twenty years before this experiment took place, a point of interest only because many texts derive Fresnel’s formula based on the results of experiment, rather than the other way around. Experimental results, within the level of error available in the mid-1800’s, are not sufficient to derive Fresnel’s formula. These results can only confirm that, within error limits, the formula provides answers consistent with experiment. In fact, Fizeau’s experimental results were so course that the only conclusion he could draw was that the displacement was less than should have been produced by the motion of the liquid. From this, he assumed the validity of Fresnel’s formula on the convection of the aether.

Fizeau’s experiment involved passing light two ways through moving water ($v \sim 7$ m/s) and observing the interference pattern obtained, as illustrated in Figure 2. The experiment was repeated by Michelson in 1886 with much more rigor, and quantitative results were obtained [5]. Working backwards from the observed fringe shift, Michelson was able to calculate an apparent convection coefficient equivalent to Fresnel’s formula. Varying the velocity and direction of the flow allowed for a variety of test points. By observing the change in interference pattern, the effective velocity of light through the moving medium, as measured in the lab frame, was calculated.

![Figure 2. The experiment of Fizeau.](image)

Within experimental limits, the results obtained by measuring the fringe shift agreed with the results predicted by Fresnel’s formula (5). However, Michelson neglected to take into account the Doppler effect of light from a stationary source interacting with moving water, and therefore concluded that the aether convection concept of Fresnel was essentially correct.

We now examine this experiment in a purely Galilean environment, taking into account the Doppler shift (change in wavelength) experienced by each beam of light. Michelson’s paper gives an excellent analysis whereby the retarded velocity, $b$, seen in the water may be considered as due to the number of collisions with atoms, the “velocity of light through the atoms,” and the width of the atoms. Since there will likely be objections to that analysis based on current understandings of the microscopic world, we present a more general approach. In what follows, the retarded velocity is again considered as due to the “collisions” (absorptions and re-emissions) of the photons in the medium, as it must be, but we do not require any assumptions as to “atom width,” or “velocity through the atom.”

Consider a tube of length $l$ filled with water running through it with a velocity $v$. The velocity of light through the water, $b$, is less than $c$. We can compare the time required to traverse the tube in each direction, and determine the shift in fringe pattern due to this time delay alone:

$$t_1 = \frac{2l}{b - v}; \quad t_2 = \frac{2l}{b + v}$$

$$\Delta t = \frac{4lv}{b^2}; \quad \Delta \phi = \frac{\Delta t \cdot c}{\lambda} = \frac{4lv\eta^2}{\lambda c}$$

Next we must consider the Doppler effects. Since the water is moving with respect to the source, the two paths of light will experience Doppler shifts upon entering the water— one a red shift, the other a blue shift:

$$\lambda_1 = (1 - v/c)\lambda$$
$$\lambda_2 = (1 + v/c)\lambda$$

While traversing the length of the tube, each light path is at a different frequency or wavelength. Two waves of unequal wavelength entering a tube in phase will arrive at the end of the tube out of phase. The shift in fringe pattern due to this effect is given by the difference in phase at the end of the tube. Thus we have:

$$\Delta \phi = \frac{2l}{\lambda_1} - \frac{2l}{\lambda_2} = \frac{2lc}{\lambda_1(c - v)} - \frac{2lc}{\lambda_2(c + v)} = \frac{4lv\eta^2}{\lambda c}$$

As will be demonstrated later, these two effects, (6) and (8) are in the opposite sense and tend to cancel each other. Thus, the total fringe shift is equal to the difference of the two effects, or:

$$\Delta N = \Delta \phi - \Delta \phi = \frac{4lv\eta^2}{\lambda c} - \frac{4lv}{\lambda c} = \frac{4lv\eta^2}{\lambda c} \left[1 - \frac{1}{\eta^2}\right]$$

(9)
This solution is identical to that obtained by Michelson in determining the extent of “aether drag.” Special relativity
derives this result by applying the relativistic velocity
dition rule. However, in so doing, relativity fails to take
into account the Doppler shift experienced by light from a
stationary source interacting with moving water. Using the
relativistic addition rule in place of \((b_n \pm v)\) would change
the last term in (9) to \((1 \mp \eta)^2\), as will be demonstrated later.

**FIZEAU AS A TEST OF SPECIAL RELATIVITY**

As stated above, in the Fizeau experiment we must
consider Doppler effects. Since the water is moving with
respect to the source, the two paths of light will experience
Doppler shifts upon entering the water. Light moving in
the opposite direction to the flow of water will be blue-shifted
\((\lambda_1)\). Light moving with the flow will be red shifted \((\lambda_2)\), as in
(7).

To see why the Doppler shift cannot be ignored in Fizeau’s
experiment, imagine the apparatus depicted in Figure 3.
All mirrors and the source are sealed in water filled
containers. The water is not flowing, but is stationary in the
containers. Alternatively, the containers could be made of
solid glass, so long as the refractive index is different than air.
The entire apparatus, with the exception of prism
(detector) P moves through the lab frame at a velocity of \(v\).
Thus, air is moving through the gap, \(l\), at a velocity of \(v\)
in the equipment frame. To first order in \(v/c\), the wavelengths
of the light detected at P are given by equation (7).

**Figure 3.** Modified Fizeau Experiment

We now fill the apparatus containers with air and pass
the entire apparatus through water as in Figure 4. In the
equipment frame, water is moving through the gap at a
velocity \(v\). As with the prism in Figure 2, the water itself
experiences a Doppler shift in the light at the interface
between the apparatus and the water, again given by
equation (7). This change in wavelength occurs at the
instant the light enters the water, and remains in effect
until the light exits the water.

**Figure 4.** Modified Fizeau Experiment II

For light that was blue-shifted at the entrance, it will be
red-shifted back to its original wavelength upon exiting the
water. For light that was red-shifted at the entrance to the
water, it will be blue-shifted back to its original value at the
exit. However, during the entire time that the light is in the
moving medium, the wavelength for one path is different
than the wavelength for the other path. If we allow light of
different wavelengths to enter a stationary medium in
phase, the two waves will be out of phase after passing a
distance through the medium, even if the two path lengths
are equal. Two unequal wavelength beams do not in general
coincide in phase, except at harmonic points. It is this effect
that is ignored in the relativistic analysis of the Fizeau
experiment.

If we, the observers, move along with the apparatus, this
setup is indistinguishable from the actual Fizeau
experiment. From our frame of reference, the equipment is
at rest, water is moving through the gap at a velocity \(v\), and
the image on the screen reflects the fringe shift due to that
motion. Thus we can replace the gap with a tube of flowing
water, hold the rest of the apparatus stationary in the lab
frame, and obtain a one-sided Fizeau experiment. This is
illustrated in Figure 5. Clearly, whatever analysis one uses
to derive the formulas for the observed fringe shift, one must
take into account the fact that the wavelength of the light in
the moving medium is different from that of the source due
to the motion induced Doppler effect of (7).

**Figure 5.** Modified Fizeau Experiment III

Thus we have two independent effects at work in the
Fizeau experiment. One shift is due to the fact that both
beams ultimately reach the screen at different times,
producing a phase shift due to the time delay. A better
description is that, in order to reach the screen at the same
time (which is where the interference occurs), the beam that
takes the least time must leave the source later. This later-
leaving beam is already out of phase by the number of cycles
late it is emitted from the source. Normally one considers
only a change in path length or a change in time, but not
both, because the wavelength or frequency of the light does
not change. However, in this case, the change in wavelength
results in the path effect, and acts in addition to (and in a
negative sense from) the time delay effect. Figure 6
demonstrates the combination of the two effects quite clearly.

In the special relativistic analysis of this experiment, the velocity of light in the moving liquid as measured in the lab frame is no longer \((b+v)\) and \((b-v)\) but is given by the relativistic velocity addition formula:

\[
b'_1 = \frac{b - v}{1 - \frac{vb}{c^2}} \quad \text{and} \quad b'_2 = \frac{b + v}{1 + \frac{vb}{c^2}}
\]

As a result, the fringe shift due to time effects alone is given as in (6) by:

\[
t_1 = \frac{2l(1-v/\eta c)}{b-v}; \quad t_2 = \frac{2l(1+v/\eta c)}{b+v}
\]

\[
\Delta t = \frac{4lv}{b^2}(1 - \frac{1}{\eta^2}); \quad \Delta \phi_t = \frac{\Delta t \cdot c}{\lambda} = \frac{4lv\eta^2}{\lambda c}[1 - \frac{1}{\eta^2}]
\]

The shift due to the unequal wavelengths in the tubes remains the same as in (8):

\[
\Delta \phi_\lambda = \frac{2l}{\lambda_1} - \frac{2l}{\lambda_2} = \frac{2lc}{\lambda_1(c-v)} - \frac{2lc}{\lambda_2(c+v)} = \frac{4lv}{\lambda c}
\]

The total fringe shift then becomes:

\[
\Delta N = \Delta \phi_t - \Delta \phi_\lambda = \frac{4lv\eta^2}{\lambda c} - \frac{4lv}{\lambda c}[1 - \frac{2}{\eta^2}]
\]

\[
(13)
\]

The Galilean (9) and special relativistic (13) results differ in the value of the last \(\eta\) term. When Michelson and Morley performed the experiment, they obtained sixty-one trials, using three different combinations of water velocity and tube length. The graph below shows the distribution of these results, normalized to a length of ten meters and a water velocity of one meter per second. The line marked RCM represents the value obtained from equation (9). The line marked SRT reflects the value obtained from (13). While there is a distribution of results, owing to experimental error, Michelson claimed an overall shift of 0.184 ± 0.02 fringe. This is completely consistent with (9), but eliminates the special relativistic result, with a value of 0.055, from consideration.

**SUMMARY**

It is very difficult to find adequate tests between special relativity and other competing theories. Most theories overlap with SRT on a vast majority of the predictions made by each, yet are based on different underlying physical principles. Ultimately one must find a test that checks not only the results of the application of the mathematical theory, but also the underlying assumptions. The major conceptual difference between SRT and most competing theories is the idea of relative simultaneity—that distant events that are simultaneous for one observer will not be simultaneous for an observer in motion relative to the first. The relativistic velocity addition rule is a direct consequence of relativistic simultaneity, and the Fizeau experiment represents a direct test of the velocity addition formula. Regardless of what the correct theory is or may be, it is clear that SRT fails to give predictions consistent with results in this experiment—an experiment performed by Michelson and Morley nine years before the introduction of special relativity.

**References:**