The Time Delay of a Solar Grazing Photon
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Introduction

We have seen in previous papers [1], [2], that moving clocks and clocks in a gravitational field slow down, not due to the effects of special relativity nor to the space-time curvature of general relativity, but due only to the principle of equivalence and the conservation of energy. However, some might argue that there has been a further “test” of the effect of gravity on time, namely the measurement of the time-delay of a round-trip, solar-grazing radar beacon performed by Shapiro in the 1960’s. In this test, Shapiro bounced a radar pulse off Mars at superior conjunction (a feat in itself for the time), and compared the measured round-trip travel time of this pulse with the expected round-trip time of a signal traveling at c for the entire trip, as determined from highly accurate planetary ephemerides. Shapiro had predicted this time-delay long before being technologically able to make such a measurement. While general relativity can be used to correctly obtain the magnitude of this delay, it is not the only explanation. As we did earlier with the analysis of clocks in motion and of clocks in a gravitational field, this paper will derive the same result without invoking the space-time curvature of general relativity.

Photon Velocity in a Gravitational Field

We begin by considering the Principle of Equivalence. Recall that, by this principle, if one is in free-fall (not being accelerated by a rocket or held in place by a floor), one cannot tell the difference between floating freely in space or falling toward a gravitating body such as the sun. No experiment one might perform could provide any knowledge about which state one is in, as the results of all experiments would be identical in both cases. This is also true of a photon in free-fall.

Now, a photon, upon entering a gravitational field, or well, acquires excess energy, compared with the energy of the surrounding gravitational field, much the way a ball dropped from a tower gains energy as it heads toward the ground. We can express the energy of this photon as:

\[ E = \frac{hc}{\kappa} \]  

Thus, an increase in energy can be viewed as an increase in effective mass, or, conversely, as a shortening or “bunching up” of the wavelength components. A visual representation of this is when smoothly flowing traffic suddenly comes upon slowing due to rubbernecking a stalled vehicle across the road. The traffic slows, and the cars which had a comfortable, even spacing now become bunched up as they pass through this area. Upon leaving the congestion, the cars once again resume their original, spread out configuration and speed.

Now, due to the principle of equivalence and its presence in the gravitational field, the photon’s velocity must slow down proportional to its decreased wavelength. For the photon, it travels in its frame of reference a distance l in a time given by l/c. If l decreases by 1/a to l/a, then the photon’s velocity will slow to c/a as well, so that it now travels the distance l/a in a time given by (l/a)(c/a), or l/c, as before. If we let the increased energy of the photon in the gravitational well (as compared with the energy of the surrounding field) be represented by aE, then we can rewrite equation (1):

\[ aE = \frac{ahc}{\kappa} = \frac{hc}{\kappa/a} \]  

From equation (2), we can again derive the new speed of this photon, realizing again that, due to the principle of equivalence, the photon’s frequency must remain unchanged:

\[ \omega = \frac{c}{\omega} = \frac{c}{\omega} \]  

Thus, the photon must slow down inversely proportional to the increase in energy it would otherwise gain during its fall through the gravitational field. We could also have simply stated that, since the velocity of a photon is equal to its frequency times its wavelength, the new velocity must be given by:
Either approach yields the same value for the new velocity of the photon. We must now turn our attention to deriving the value of the factor \( a \).

If a hydrogen atom is carried into a gravitational well, it will generate or absorb energy at a lower frequency than it would outside the field. Thus, if we build a clock based on the frequency absorbed or emitted by this atom, it will run more slowly than an identical clock not inside the field. The rate of slowing of this clock will be the inverse of the equation used to derive the gravitational blue-shift for a photon falling into a gravitational well. If \( n' \) represents the frequency generated by the clock inside the gravitational field, and \( n \) the frequency generated far removed from that field, we have the following equation for the clock slowing:

\[
Q = \left(1 + \frac{GM}{c^2 R}\right)^{-1} \cdot \frac{c}{c'}
\]  

If this clock measured the frequency of a photon generated far outside the field, the measured frequency would be blue shifted by the inverse of the above factor. This is graphic evidence that the frequency of the photon in free-fall has not changed. We know that the clock in the gravitational field runs slowly by the relation in equation (5), because this has been tested many times with actual cesium clocks. We also know that the blue-shift of a photon in free fall as measured by these same clocks is exactly the inverse of equation (5), as the Pound-Rebka-Snyder experiment (1960, for example) has shown. Now, if the photon itself were actually blue-shifted, the measured shift would be twice that given by the inverse of (5). First, the photon would be blue shifted by this amount, and this blue shifted frequency would appear to be shifted even higher when measured with the slow running clock. But the effect appears only once, and it is due only to the clock slowing—the frequency of the photon, due to the principle of equivalence, has remained unchanged.

So now we have the new velocity of a photon in free-fall through a gravitational field. Combining equations (5) and (4), we get:

\[
c_{\text{c}}^{(0)} = c \left(1 + \frac{GM}{c^2 R}\right)^{-1}
\]  

However, equation (6) must be modified slightly. The velocity of the photon is constantly changing during its trip, in a manner always proportional to \( 1/r \), \( r \) being the distance from the sun at any given point. We therefore replace \( R \), the grazing radius of the sun, with \( r \), the instantaneous distance from the sun at any time along the photon’s journey:

\[
c_{\text{c}}^{(0)} = c \left(1 + \frac{GM}{c^2 r}\right)^{-1}
\]  

This is still not the entire story. Equation (7) represents the velocity of the photon as measured locally by a clock traveling with the photon (in the vicinity of the sun). However, we are interested in the value of this velocity as measured using an Earth based clock. Clearly, a velocity measured with a standard clock will be less than the same velocity measured with a gravitationally slowed clock. Taking the Earth based clock to be our standard, we see that the velocity as measured by that clock will be even slower than the velocity as measured by a local clock carried with the photon. The additional scaling of the velocity will be identical to the scaling already provided in (7). The effect appears twice. Once due to the actual slowing of the photon in its own reference frame measured by its own clocks, and again to translate this velocity to the velocity as measured by an Earth based clock. The velocity as measured by an Earth based clock, omitting terms of order higher than \( 1/c^2 \), is then given by:

\[
c_{\text{c}}^{(0)} = c \left(1 + \frac{GM}{c^2 r}\right)^{-1} \left(1 + \frac{GM}{c^2 r}\right)^{-1} \cdot c \left(1 + \frac{2GM}{c^2 r}\right)^{-1}
\]  

**The Derivation of the Time Delay**

From the figure, and from equation (8), the following relations can be derived, where \( T \) is the total time of the photon’s travel from 0 to \( d \), as measured by an Earth based clock:

\[
T = \frac{d}{c_{\text{e}}^{(0)}}
\]
In equation (13), we can obtain the effective path length of a photon traveling at $c$ for a time $T$ by multiplying through by $\frac{1}{c}$. Doing this, and breaking the integral into parts, yields the effective path length of the photon:

$$\int_0^T dt = \int_0^T \frac{ds}{c(1 + \frac{2GM}{c^2 r})^{-1/2}} = \frac{1}{c} \int_0^T \frac{ds}{(1 + \frac{2GM}{c^2 r})}$$

(10)

$$T = \int_0^T \frac{ds}{c} = \frac{1}{c} \int_0^T (1 + \frac{2GM}{c^2 r}) ds$$

(11)

$$ds = (r^2 - R^2)^{-1/2} r dr$$

(12)

$$T = \frac{1}{c} \int_0^T \frac{d^2 + R^2}{R} \left(1 + \frac{2GM}{c^2 r}\right) (r^2 - R^2)^{-1/2} r dr$$

(13)

In equation (13), we can obtain the effective path length of a photon traveling at $c$ for a time $T$ by multiplying through by $c$. Doing this, and breaking the integral into parts, yields the effective path length of the photon:

$$d^{\text{eff}} = cT = \frac{1}{c} \int_0^T \frac{d^2 + R^2}{R} (r^2 - R^2)^{-1/2} r dr + \frac{1}{c} \int_0^T \frac{d^2 + R^2}{R} \left(1 + \frac{2GM}{c^2 r}\right) (r^2 - R^2)^{-1/2} r dr$$

(14)

$$d^{\text{eff}} = (r^2 - R^2)^{1/2} + \frac{2GM}{c^2} \ln \left\{ \frac{r}{(r^2 - R^2)^{1/2}} \right\} \left|_{\frac{d^2 + R^2}{R}} \right.$$ 

(15)

$$d^{\text{eff}} = d + \frac{2GM}{c^2} \left\{ \ln \left( \sqrt{d^2 + R^2} + d \right) - \ln(R) \right\} = d + \frac{2GM}{c^2} \ln \left( \frac{\sqrt{d^2 + R^2} + d}{R} \right) \bullet d + \frac{2GM}{c^2} \ln \left( \frac{2d}{R} \right)$$

(16)

In equation (16), the last approximation holds for $d \gg R$. The last term on the right-hand side of equation (16) represents the effective increase in path length, and is given by:

$$\Delta d = \frac{2GM}{c^2} \ln \left( \frac{2d}{R} \right)$$

(17)

For distances on the order of Earth or Mars, $\Delta d$ is approximately 19 km. Thus, for a light signal traveling from Earth to Mars and back, with Mars at superior conjunction, the round trip increase in effective path length due to the time-delay of the photon is approximately 76 km, resulting in an increase in round trip travel time of 250 microseconds (out of a total trip on the order of thirty minutes). This result is equivalent to the results of relativity theory, and is confirmed (Shapiro, et al) to an accuracy of 0.1%.


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